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CONCEPT OF DUMMY SPACE IN THE STUDY OF FRACTIONAL FACTORIAL DESIGN WITH MINIMUM ABERRATION

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ABSTRACT

An important application of statistical method to industrial research is in the design and analysis of experiments in connection with the improvement of manufacturing processes. The factional factorial designs are most economical in this scenario. Therefore, an attempt is made to investigate the effect of increase in the dummy space (value) in fractional factorial design so that more information about the interior of the experimental region can be extract. Some examples are also present in the study to show the variation in the treatment effect and in error degree of freedom with increase in the dummy value.

KEYWORDS: Resolution of Design, ANOVA, Dummy Values, Factor Interaction, Etc

INTRODUCTION

Scientists around the world do a variety of activities involving developing new products, improving designs and ongoing manufacturing process. Such type of goals can be achieved through experimental design theory. It consists of a series of tests in which purposeful changes are made to the input factors of a product. When these factors become four or more and if the experimenter can assume that certain high-order interaction are negligible, the number of treatments can reduce by running a fraction of the complete factorial experiments. These designs are called fractional factorial designs and are among the most widely used types of designs for product and process design and for process trouble shooting. Box and Hunter (1961) the first approach the problem by introducing the notation as a goodness criterion for designs. Since designs of the same resolution may not be equally good. Fries and Hunter (1980) suggest the minimum aberration criterion to further discriminate designs. The minimum aberration criterion was already used implicitly in the construction of designs in the class work at the National Bureau of Standards in 1957-1959. A design is of resolution R if number C-factor effect is confounding with any other effects containing less than $R-C$ factors. A design 2_{III}^{n-k} resolution does not confound main effects with one another but does main effects with two-factor interactions and a design 2_{IV}^{n-k} resolution does not confound main effects with two-factor interactions but does confound two–factor interactions with one-another. A design 2_{V}^{n-k} resolution in which no main effect or two–factor interaction is confounded with any other main effect or two–factor interaction but two factor interactions are confounded with three-factor interactions. The resolution of a two-level fractional factorial design is the length of the shortest word in the defining relation. Usually an experimenter will prefer to use a design which has the highest possible resolution.

The best fractional factorial design is the most economical one while enabling satisfactory estimation of the effects of interest. Therefore, in present study, an attempt is made to investigate the effect of increase in the dummy space (value) in fractional factorial design so that more information about the interior of the experimental region can be extract.

Some examples are also present in the study to show the variation in the treatment effect and in error degree of freedom with increase in the dummy value.

MINIMUM ABERRATION CRITERION

Box and Hunter (1961) first approach the problem by introducing the notion of resolution as a goodness criterion for designs, but designs of the same resolution may not be equally good. Fries and Hunter (1980) suggest the minimum aberration criterion to further discriminate designs. The purpose of minimum aberration is to provide a method for selecting a best subsets of designs from the set of 2^{n-k} fractional factorial designs of highest resolution 'best' is defined in terms of the concept of aberration. For example, in semiconductor fabrication plant, an experiment is rum with two-level fractional factorial designs in $N = 8$ runs where five factors each at two levels are studied.

A =Aperture Setting (Small, Large)

B =Exposure Time (20% below nominal, 20% above nominal)

C =Development Time (30 sec., 45 sec.)

D = Mask Dimensions (Small, Large)

E = Etch Time (14.5min., 15.5min.)

If an experimenter has prior knowledge concerning the possible importance of certain main effects and interactions

$D = AB\ E = AC$

$D = AC\ E = BC$

$D = ABC\ E = ACD$

Here the more common situation is consider in which prior knowledge diffuse concerning the possible greater importance of certain specific main effects relative to others. It is also assume that the experimenter believes initially that main effects are more important than two factor interactions, that two factor interactions are more important than three factor interactions and so on. Therefore, the three different designs are

Design: a

Table 1

$D = AB, E = AC$	
$I = ABD = ACE = BCDE$	
A	$BD = CE = ABCDE$
B	$AD = CDE = ABCE$
C	$AE = BDE = ABCD$
D	$AB = BCE = ACDE$
E	$AC = BCD = ABDE$
BC	$DE = ACD = ABE$
CD	$BE = ABC = ADE$

Design: b

Table 2

$D = AC, E = BC$	
$I = ACD = BCE = ABDE$	
A	$CD = BDE = ABCE$
B	$CE = ADE = ABCD$
C	$AD = BE = ABCDE$
D	$AC = ABE = BCDE$
E	$BC = ABD = ACDE$
BD	$AE = ABC = CDE$
AB	$DE = BCD = ACE$

Design: c

Table 3

$D = ABC, E = ACD$	
$I = BE = ABCD = ACDE$	
A	$BCD = CDE = ABE$
B	$E = CD = AB = AE = ACD = BCDE = ABCDE$
C	$ABD = ADE = BCE$
D	$ABC = ACE = BDE$
AC	$BD = DE = ABCE$
BC	$AD = CE = ABDE$

$R - III$ Is the maximum attainable resolution for 2^{5-2} designs therefore considering design $(a), (b)$ and (c) are of resolutionIII. For the designs of resolution III un-confounding estimates are obtained for all main effects if one can assume that three factor and higher order interactions are negligible. Tables provide a summary of these designs with regard to confounding among two factor interactions and main factor. Un-confounding estimates are obtained for all two-factor interactions not shown. There are greater amount of confounding in (c) and less in (a)& (b). But (c) doesn't fully satisfy the condition of resolution criterion (i.e. main effects doesn't confound the main effects), so the design (a)& $(b)$are appropriate.

The word lengths in the defining relation for design (a) and (b) is (3,3,4), (3,3,4) and (c) is (2,4,4) the defining relation for design (c) has only one two-word and two four-word length whereas (a) & (b)have only one four and two three-word length. Thus we can't be used the design(c) design, it satisfy the minimum word length criterion but not satisfied the confounding pattern of minimum aberration, (a) & (b) designs which minimizes the number of words in the defining relations that are of minimum length, therefore these designs are called minimum aberration designs. When comparing two designs using resolution as the criterion, one can considered the lengths of the shortest word in each defining relations. If these designs (a)& (b) are equal the two designs is regard as being equivalent with aberration as the criterion, however one continuous to examine the length of next shortest word in each defining relation until one designs is ranked one superior to the other.

Given that resolution is maximized and equal to R_{max}, minimizing aberration ensures that a design has the minimum number of word of lengthsR_{max} , which in term means that smallest number of main effects will be confounding with the interactions of order$R_{max} - 1$, the smallest number of main effects will be confounding with the interactions of order $R_{max} - 2$ and so on. The concept of aberration is a natural extension of resolution.

GENERALIZATION

To generalize the setup a 2_R^{n-k} design is constructed by the first writing down a full two-level fractional design in$n - k$ factors and then defining the column vectors for p additional factors by associating them with certain interaction column involving the first $n - k$ factors. Each such assignment results in a generator equal to the identity$I$.

Taking the products of the k generators one at a time, two at a time etc., gives the defining relation which has 2^{k-1} words plus$I$. For fixed N and k, p the problem is to select the best 2_R^{n-k} design.

Suppose two 2^{n-k} design (s) and (t) for maximum resolution R_{max} are to be compared and their defining relations have their word - length patterns.

(s)$\{R_{max}^{s_0}(R_{max}+1)^{s_1}(R_{max}+2)^{s_2} \dots \dots \dots (R_{max}+m)^{s_m}\}$

(t)$\{R_{max}^{t_0}(R_{max}+1)^{t_1}(R_{max}+2)^{t_2} \dots \dots \dots (R_{max}+n)^{t_n}\}$

Determine the first subscript $i$such that $s_i \neq t_i$ if $s_i < t_i$ then design (s) is the better design: otherwise t is the better design. We consider designs for fixed N and Kthat result from these procedure designs of minimum aberration. We will consider how this principal can be employed in practice to construct useful designs. The National Bureau of Standard tabulation of two –level fractional factorial designs[Connor and Zelen 1959] which makes use of a similar criterion, indicates in this statement one of the kinds of problems that must be addressed-"Although considerable efforts we made to find solutions which have the maximum number of two-factor interactions confounding with three-factor and higher-order interactions, other solutions may exist having a larger number of measurable two-factor interactions." criteria related problem are suggested by Addelman (1969).

CENTER VALUE FRACTIONAL FACTORIAL DESIGNS

Box, Hunter and Hunter (1978p 410) provides a useful catalogue of two level fractional factorial designs with minimum aberration. Franklin (1984) constructs more minimum aberration designs. Chen and Wu (1991) and Chen(1992) investigate some theoretical properties of minimum aberration designs. Zhang and Park (1999) the minimum aberration criterion is extended for choosing blocked fractional factorial designs with respect to both treatment and blocks. The following table gives the list of fractional design.

Table 4

Number of factors	8 run	16 run	32 run	64 run
3	$FF\,3 - 08$	$--$	$--$	$--$
4	$FF\,4 - 08$	$FF\,4 - 16$	$--$	$--$
5	$FF\,5 - 08$	$FF\,5 - 16$	$FF\,5 - 32$	$--$
6	$FF\,6 - 08$	$FF\,6 - 16$	$FF\,6 - 32$	$--$
7	$FF\,7 - 08$	$FF\,7 - 16$	$FF\,7 - 32$	$FF\,7 - 64$
8	$--$	$FF\,8 - 16$	$FF\,8 - 32$	$FF\,8 - 64$
9	$--$	$FF\,9 - 16$	$FF\,9 - 32$	$FF\,9 - 64$
10	$--$	$FF\,10 - 16$	$FF\,10 - 32$	$FF\,10 - 64$
11	$--$	$--$	$FF\,11 - 32$	$FF\,11 - 64$

Let $D(2^{n-k}, 2^p)$ denote a 2^{n-k} design in 2^pblock of size $2^{n-k-p}(p < n-k)$, it can be viewed as a $2^{(n+p)-(k+p)}$ fractional factorial design, where the factors are divided into different types; n treatments factors $1, 2, 3, 4 \dots\dots. n$ and p block factors $b_1, b_2, b_3, \dots\dots\dots.., b_p$. The 2^k combinations of the block factors are used to divide the 2^{n-k} treatment combinations into 2^p blocks. In such design, there are two types of words, which are called respectively treatment defining words and block defining words.

Center value means a dummy value (space) including in the experimental design to know the effects of the treatment (variable). Center value are recommended for most of design - 8 run design with 3-4 factors, 16 run design with 4-8 factors, 32run designs with 6-12 , 64 run designs with 8-16 factors and so on. Center values provide information about the interior of the experimental region. We apply the treatments on the experimental area. The effects of the treatments are not openly shown because of the neighbor effects of the treatments those apply surrounding areas or experimental field.

For example, a chemical manufacturing plant, factorial experiment is carried out in the pilot plant to study the factors which influence the filtration rate of the product. The four factors are temperature A, pressure B, concentration of formaldehyde C, and stirring rate D, each factor is present at two-levels. Therefore 2^{4-1}center value fractional factorial design with two blocks is given by

Table 5

Run	A		B		C		D=ABC		Filter. Rate	
	Block 1	Block 2	Block 1	Block 2	Block 1	Block 2	Block 1	Block 2	Block 1	Block 2
1	-1	-1	-1	-1	-1	-1	-1	1	45	43
2	1	1	-1	-1	-1	-1	1	-1	100	71
3	-1	-1	1	1	-1	-1	1	-1	45	48
4	1	1	1	1	-1	-1	-1	1	65	104
5	-1	-1	-1	-1	1	1	1	-1	75	68
6	1	1	-1	-1	1	1	-1	1	60	86
7	-1	-1	1	1	1	1	-1	1	80	70
8	1	1	1	1	1	1	1	-1	96	65
9	0	0	0	0	0	0	0	0	-	-
10	0	0	0	0	0	0	0	0	-	-
11	0	0	0	0	0	0	0	0	-	-
12	0	0	0	0	0	0	0	0	-	-

The ANOVA table for such experiment shows the different in the treatment sum of square with inclusion of different center values.

ANOVA Table

Table 6

Source	Degree of Freedom			Sum of Square			M. S. Square			Variance ratio		
		2 Center values	4 Center values		2 Center values	4 Center values		2 Center values	4 Center values		2 Center values	4 Center values
Treatment	7	9	11	3650.4375	19358.45	29830.458	521.49107	2150.93889	2711.85985	**1.7609**	**9.3300**	**14.3729**
Block	1	1	1	7.5625	6.05	5.0416	7.5625	6.05	5.0416	0.025537	0.026247	0.026720
Error	7	9	11	2072.9375	2074.45	2075.4584	296.1339	230.4944	188.6780			
Total	**15**	**19**	**23**	**5730.9375**	**21438.95**	**3910.9583**						

In another example, an experiment was performed in a semiconductor manufacturing plant to study the effect of six factors on the curvature or camber of the substrata devices produced. The six variables and their levels are shown below

Table 7

Level	Lamination Temperature	Lamination Time	Lamination Pressure	Firing Temperature	Firing Cycle Time	Firing Dew-Point
-	55	10	5	1580	17	20
+	75	25	10	1620	29	26

Therefore 2^{6-1} Center value fractional factorial design with two blocks is given by

Table 8

Run	A		B		C		D		E		F=ABCDE		Mean Effect	
	Block 1	Block 2	Block 1	Block 2	Block 1	Block 2	Block 1	Block 2	Block 1	Block 2	Block 1	Block 2	Block 1	Block 2
1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	281	282
2	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	-1	285	284
3	-1	-1	1	1	-1	-1	-1	-1	-1	-1	1	-1	284	283
4	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	1	287	288
5	-1	-1	-1	-1	1	1	-1	-1	-1	-1	1	-1	283	282
6	1	1	-1	-1	1	1	-1	-1	-1	-1	-1	1	285	286
7	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	1	284	285
8	1	1	1	1	1	1	-1	-1	-1	-1	1	-1	288	287
9	-1	-1	-1	-1	-1	-1	1	1	-1	-1	1	-1	288	287
10	1	1	-1	-1	-1	-1	1	1	-1	-1	-1	1	291	291
11	-1	-1	1	1	-1	-1	1	1	-1	-1	-1	1	290	290
12	1	1	1	1	-1	-1	1	1	-1	-1	1	-1	294	293
13	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	1	288	289
14	1	1	-1	-1	1	1	1	1	-1	-1	1	-1	293	292
15	-1	-1	1	1	1	1	1	1	-1	-1	1	-1	292	291
16	1	1	1	1	1	1	1	1	-1	-1	-1	1	294	295
17	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	-1	284	283
18	1	1	-1	-1	-1	-1	-1	-1	1	1	-1	1	286	287
19	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	1	285	286
20	1	1	1	1	-1	-1	-1	-1	1	1	1	-1	290	289
21	-1	-1	-1	-1	1	1	-1	-1	1	1	-1	1	284	285
22	1	1	-1	-1	1	1	-1	-1	1	1	1	-1	288	287
23	-1	-1	1	1	1	1	-1	-1	1	1	1	-1	287	286
24	1	1	1	1	1	1	-1	-1	1	1	-1	1	289	290
25	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	1	289	290
26	1	1	-1	-1	-1	-1	1	1	1	1	1	-1	294	293
27	-1	-1	1	1	-1	-1	1	1	1	1	1	-1	293	292
28	1	1	1	1	-1	-1	1	1	1	1	-1	1	297	296
29	-1	-1	-1	-1	1	1	1	1	1	1	1	-1	291	290
30	1	1	-1	-1	1	1	1	1	1	1	-1	1	294	295
31	-1	-1	1	1	1	1	1	1	1	1	-1	1	293	294
32	1	1	1	1	1	1	1	1	1	1	1	-1	297	296
33	0	0	0	0	0	0	0	0	0	0	0	0	-	-
34	0	0	0	0	0	0	0	0	0	0	0	0	-	-
35	0	0	0	0	0	0	0	0	0	0	0	0	-	-
36	0	0	0	0	0	0	0	0	0	0	0	0	-	-
37	0	0	0	0	0	0	0	0	0	0	0	0	-	-
38	0	0	0	0	0	0	0	0	0	0	0	0	-	-
39	0	0	0	0	0	0	0	0	0	0	0	0	-	-
40	0	0	0	0	0	0	0	0	0	0	0	0	-	-

The ANOVA table for such experiment shows the different in the treatment sum of square with inclusion of different center values and given by

ANOVA Table

Table 9

Source	Degree of Freedom			Sum of Square			M. S. Square			Variance ratio		
		8 Center values	10 Center values		8 Center values	10 Center values		8 Center values	10 Center values		8 Center values	10 Center values
Treatment	31	39	41	176.75	1069683.2	1273227.29	34.7338	27427.7744	31054.4341	**73.0008407**	**72277.2594**	**85973.3510**
Block	1	1	1	0.25	0.20	0.19	0.25	0.20	0.19	0.52543085	0.527037	0.5260098
Error	31	39	41	14.75	14.8	14.81	0.4758	0.37948	0.36121			
Total	63	79	83	1091.75	1069698.2	1273242.29						

CONCLUSIONS AND DISCUSSIONS

Just as fractional designs are useful in wide variety of application fields, so are fractional factorial designs. Although, it is assumed that they are associate with social and physical sciences, but they also extensively used in manufacturing industry. The primary advantage of fractional factorial designs is they permit effects that may be significant to be estimated with a reasonable number of runs. Resolution III designs are frequently used in the first stage of experimentation for the purpose of identifying factors that seem important and resolution IV and V on the second stage. In the present study, these designs are studied with minimum aberration criterion in the presence of dummy values. It is observed that whenever the center values include in the experimental area it creates the distance between the blocks or the area of treatment. In this condition different types of treatments which are applied in the blocks are not affects each-other simultaneously orin simple words the variation between the treatments is openly shown. Center values provide additional degree of freedom for error which results in greater power when testing the significance. Due to these reasons the recommended center points should typically be included in the experiment. Under certain circumstances it is reasonable to delete recommended center points from designs; including center points will never hurt the statistical properties of a properly analyzed experiments. According to the requirements of the information, experimenter caninclude or reduce the center value because it not affects the basic structure or information of designs. Center values in the fractional factorial designs not affect the treatment effects but show the variation in the treatments sum of square or error sum of square and increase the degree of freedom for errors.

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